Lecture 1

Quantum Dots: Coulomb blockade, tunneling, cotunneling
Outline of the course:

• Quantum Dots
• Kondo effect in nano-devices
• From Fermi liquid to Luttinger liquid

For reading:

Transport through QDs: W.G. van der Wiel et al, RMP 75 (2003)
SET and Coulomb blockade: M.A. Kastner, RMP 64 (1992)
Popular reading: Leo Kouwenhoven and Charles Marcus, Physics World 1998
See also in the web Lecture courses of Ya. Blanter, Y. Gefen, Yu. Galperin

* Some transparencies are courtesy of Yuval Gefen, Yaroslav Blanter and Yuri Galperin
Connection with other lectures:

This Lecture Course

Robert Shekhter:
Mechanically assisted single-electronics
Quantum coherent nano-electromechanics

Vladimir Kravtsov
Quantum dots and random matrices

Peter Fulde:
Electron correlations on nano- and subnano scale
Outline of this lecture

• What is Nano?

• Examples of QD:
  • Vertical vs lateral QDs
  • Diffusive vs ballistic QDs
  • Metallic vs semiconductor QDs
  • Open vs close QDs
  • Coulomb blockade
  • Sequential tunneling
  • Elastic vs inelastic cotunneling
  • “Universal” Hamiltonian
Characteristic scales in nanoscience

$1 \text{ nm} = 10^{-9} \text{ m}$

Atoms  Molecules & Clusters  Electron mean free path  Bulk materials

Micro  Meso  Macro

Modern electronic devices belong to mesoscopic scale
Nano means Big!? 

Nanoscale objects do not fully belong to the microcosm

Many atoms, electrons, etc., are involved

Number of degrees of freedom is large

Micro  Meso  Macro

Nuclei  Nano-objects  Fluids
Atoms  Crystals
Small molecules  Glasses
CMOS TECHNOLOGY

Intel’s Prescott processor (released March 2004):
- 150 million transistors
- 90 nm design rules
- 3.4 GHz clock frequency

DRAM chips:
- 4 Gb chips demonstrated
  \[ \sim 10^9 \text{ transistors/cm}^2 \]
Two Dimensional Electron Gas (2DEG)
Quantum dots

- Tune: gate potentials, temperature, field…
- Measure: I-V curves, conductance G…
- Aharonov-Bohm interferometry, dephasing, coherent state manipulation…
Quantum dots: from simple to complex


J. P. Kotthaus (1995)


H. Jeong et al (2001)

Self-assembled quantum dots are periodic arrays of "artificial atoms". They are considered to be promising systems for heterostructure lasers.
Nanoelectromechanical shuttling: QD devices

Cerocene $\text{Ce}\left(C_{8}H_{8}\right)_{2}$ Ytterbocene

$\text{Ce(COT)}_{2}$

$\text{Cp}^*_{2}\text{Yb(bipy)}$

COT = C8H8

$\text{Cp}^* = \text{C5Me5}$, $\text{bipy} = (\text{NC5H4})_2$
Molecular Transistor
Fullerenes

H.Park et al, Nature 2000
Transition metals inside fullerens

(A)

(B)

(a)
Carbon Nanotubes

\[ \frac{dE}{dk} < 0 \quad \frac{dE}{dk} > 0 \]
Nanotube peapods: $C_{60} @ CNT$
Realization of lateral QD in 2DEG

2DEG reservoir

Plunger; gate modulating number of electrons

Plunger Gate

confining gate

electrons puddle

2DEG reservoir

tunnel barrier

$V_P$

$U_c$
Characteristic parameters:

size: \(100 \text{ Å} \rightarrow 2 \text{μm}\)

# electrons: \(0 \rightarrow \text{hundreds}\)

mobility

(of 2DEG in strong magnetic fields)
original Integer Quantum Hall Effect
current world record (Weizmann)

\( \sim (30-50) \cdot 10^3 \text{ cm}^2 / V \cdot \text{sec} \)

\( \sim 36 \cdot 10^6 \)

CONTROL:

- size of QD
- density of electrons \(\rightarrow\) # electrons
- (mobility; disorder)
- shape
- contact to leads
Vertical QDs

advantages: easy access to small # electrons, symmetric QDs

disadvantages: hard to control shape/size; dot-lead coupling
Metallic QDs

- **Size**: 30Å and up
- **λ_F**: a few Å
- **# Electrons**: > many hundreds

*Originally: statistics of an ensemble*
*Today: can attach leads to a single QD*
*Little control: QD-lead coupling; size of QD*
*Special appeal: QDs with special properties: SC; magnetic…*
NON INTERACTING ELECTRONS

diffusive vs. ballistic

**diffusive**

\[ E_{th} = \frac{\hbar}{(\text{diffusion time})} = \frac{\hbar}{(L^2 / D)} \]

**ballistic**

\[ E_{th} = \frac{\hbar}{(\text{time of flight})} = \frac{\hbar}{(L / v_F)} \]

**dirty ballistic**

Time of flight ≠ Thouless energy

(Altland, Gefen, Montambaux)
“Metallic” vs. discrete spectrum

\[ \Delta \ll kT \]

\[ \Delta \gg kT \]

\[ kT \]

\[ \Delta \]

\[ kT \]
Open vs. Closed

\[ \frac{\Gamma_c}{\hbar} = \text{decay rate of a QD level into channel } c \]

Total level width = \( \Gamma = \sum_c \Gamma_c \)

closed QD (charge on the dot is nearly quantized)

\[ \Gamma < E_c \]
NON INTERACTING ELECTRONS

energy measured from $\mu$

$E_F$

$\hbar/\tau_{el}$

$E_{th}$

$\Delta$

$0$

non-perturbative

diagrammatics

diffusive approximation (diffusons, Cooperons)

universal (RMT) regime

$\Delta < \omega < E_{th}$

universal

diagrammatics, universal
Metallic quantum dots: many-electron system

Random Matrix Theory

GOE  GUE

Wigner-Dyson statistics

Orthogonal (GOE)

$P\left(\{E_\mu\}\right) \propto \exp\left(\frac{\beta}{2} \sum_{\nu \neq \mu} \ln \left| \frac{E_\mu - E_\nu}{\delta} \right| \right)$

$\beta = 1$

Unitary (GUE)

$\beta = 2$

Simplectic (GSE)

$\beta = 4$

= artificial atom
Coulomb blockage

Ivar Giaever

1973

To move an electron to a confined region one has to pay for its repulsion from existing electrons.
The principle of the Coulomb blockade

Why R matters?
- time delay $\delta t = eR/V$
- duration $\tau \sim \hbar/eV$

$\delta t \gg \tau \rightarrow R \gg \hbar/e^2$

Energy stored is $q^2/2C$

Because of environment capacitances it is difficult to observe CB in single junctions

At $|q| < e/2$ the electron tunneling will increase the energy stored in the barrier - one has to pay for the tunneling by the bias voltage.
Coulomb blockade

Repulsion at the dot

Attraction to the gate

\[ E = QV_g + \frac{Q^2}{2C} \]

\[ V_g = -\left(N + \frac{1}{2}\right) \frac{e}{C} \]

At the energy cost vanishes!

Single-electron transistor (SET)
How many electrons?

Shifts potential
= induces charge
$C_g V_g$

$E = E_C \left(N - \frac{C_g V_g}{e}\right)^2$

$N = \left[\frac{C_g V_g}{e} + \frac{1}{2}\right]$
Coulomb staircase

Nonlinear transport: upon increasing V more charging states become available

-2 ≤ N ≤ 2
-1 ≤ N ≤ 1

Ohm’s law: I = GV

\[ V_{ih,n} = \frac{(2n+1)e}{C_\Sigma} \]

\[ Q_g = 0 \]
LATERAL QDs: possible parameters

Temperature $< 1$ K (as low as 10-30 mK)
elastic mean free path $\approx 1$ - 150 $\mu$m

$n_s \approx 10^{11}$ - $10^{12}$ cm$^{-2}$

$E_F \approx 10$ - 20 meV

$\lambda_F \approx 50$ nm

# electrons: 0 - hundreds

single particle level spacing $= \Delta \sim (0.01 \text{ meV} \sim 0.1 \text{ K})$

Thouless energy $= E_{th} \sim (0.3 \text{ meV} \sim 3 \text{ K})$

charging energy $= E_c \sim (1 \text{ meV} \sim 10 \text{ K})$
THE “UNIVERSAL” HAMILTONIAN

\[ H = H_{sp} + H_{\text{int}} \]

\[ H_{\text{int}} = \frac{1}{2} \sum_{\alpha, \beta, \gamma, \delta} H_{\alpha \beta \gamma \delta} \hat{\Psi}^{\dagger}_{\alpha \sigma_1} \hat{\Psi}^{\dagger}_{\beta \sigma_2} \hat{\Psi}_{\gamma \sigma_2} \hat{\Psi}_{\delta \sigma_1} \]

\[ H_{\alpha \beta \gamma \delta} = \int dr_1 dr_2 V(r_1 - r_2) \phi_\alpha (r_1) \phi_\beta (r_2) \phi^*_\gamma (r_2) \phi^*_\delta (r_1) \]

Note: only orbital indices (no spin-orbit)

\[ H_{\text{int}} = H_{\text{int}}^{(0)} + H_{\text{int}}^{(1/g)} \]

universal \hspace{1cm} non-universal, fluctuating
CHARGING HAMILTONIAN

\[ H = H_{sp} + H_{\text{int}} \]

\[ H_{\text{int}} = H^{(0)}_{\text{int}} + H^{(1/g)}_{\text{int}} \]

\[ H^{(0)}_{\text{int}} \Rightarrow E_c \left( \hat{n} - N_0 \right)^2 \]

\[ \rightarrow E_c \left( \sum \Psi^\dagger_\alpha \Psi_\alpha \right) \cdot \left( \sum \Psi^\dagger_\beta \Psi_\beta \right) - 2E_c N_0 + E_c N_0^2 \]
Metallic Quantum Dot: Universal Hamiltonian

Metallic grain or small island of electron gas

Electron-electron interactions in isolated metallic grains

- Mean-level spacing: $\Delta = \langle E_{\alpha+1} - E_{\alpha} \rangle$ (kinetic energy)
- Thouless energy: $E_T \sim D \cdot L^{-2}$ (diffusive regime)
- $E_T \sim v_F L^{-1}$ (ballistic regime)

$g = \frac{E_T}{\Delta} \gg 1$ metallic grain

$H_0 = \sum_{\alpha} E_{\alpha} n_{\alpha}$

$E_c = \frac{e^2}{2C}$

$H_{\text{int}} = E_c (\hat{n} - N)^2 - J (\hat{S}^z)^2 - \lambda_{\text{BCS}} \sum_{\alpha} \hat{T}^\dagger \hat{T}$

\[ \hat{n} = \sum_{\alpha,\sigma} d_{\alpha\sigma}^+ d_{\alpha\sigma} \]

$S^y = \frac{1}{2} \sum_{\alpha,\sigma,\sigma'} d_{\alpha\sigma}^+ \sigma_{\sigma\sigma'} d_{\alpha\sigma'}$

$T = \sum_{\alpha} d_{\alpha\uparrow} d_{\alpha\downarrow}$

Superconducting

**Coulomb blockade**


Aleiner, Brouwer, Glazman (2002)

Short-range interaction

$E_c = 4 |J| \sim \Delta$

Scaling:

Coulomb interaction

$E_c = r_s (k_F L)^{d-1} \Delta \gg |J|$
TRANSPORT THROUGH a QD: thermally activated conduction

\[ H = H_{sp} + E_c (\hat{n} - N_0)^2 + H_{\text{leads}} + H_{\text{tunneling}} \]

\[ H_{\text{leads}} = \sum \xi_k c_k^\dagger c_k \quad \text{for each lead} \]

\[ H_{\text{tunneling}} = \sum_{\alpha,k,n,s} t_{\alpha n} c_{\alpha k s}^\dagger d_{ks} + h.c. \]
Tunneling in metals (No CB)

- Tunnel barrier

- Tunnel current: from occupied to empty

\[ I = G_T V \]

- Rate:

\[ \Gamma = \frac{I}{e} = \frac{G_T}{e^2} eV \]
Tunneling and Coulomb blockade

- Now the same with Coulomb blockade
- Electrostatic energy must be paid

\[ eV < \Delta E_c : \Gamma = 0 \]

- blockade

\[ eV > \Delta E_c : \Gamma = \frac{G_T}{e^2} (eV - \Delta E_c) \]
Inelastic cotunneling

inelastic cotunneling

(excitations left behind)

state $k$ on Left $→$
state $k′$ on Right + Dot (n filled; m empty)
Elastic cotunneling

elastic cotunneling

(no excitations left behind)
Tunneling and co-tunneling (summary)

Electron-like co-tunneling

Hole-like co-tunneling

Elastic

Inelastic
DISCRETE SPECTRUM QD: coherent vs. Incoherent transport

\[ H = H_L + H_R + H_{dot} + H_{tun} \]

\[ H_{dot} = \varepsilon \sum_{\sigma} n_{\sigma} + E_c n_\uparrow n_\downarrow \]

\[ H_{tun} = t_\alpha c_\alpha^+ d_\sigma + h.c. \]

This is a story for next lecture!